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Abstract

Despite the theoretical effectiveness of carbon taxes as an instrument of climate policy, political constraints still halt their more common adoption. Policymakers thus may need to implement climate policy via existing policy levers that are not explicitly labelled as carbon pricing. In this paper we evaluate a novel - potentially politically feasible - approach of conducting climate policy through the pension system. While typically policymakers grant tax relief on all pension savings, we suggest that the relief could be granted only on "green" savings. To model the policy, we rely on the Diamond-type overlapping generations framework. We find that, conditional on the unconstrained optimal policy implementing a 2°C temperature rise, our constrained optimal policy implements a 2.3°C temperature rise.

1 Introduction

The scientific consensus about the impact of anthropogenic greenhouse emissions and predictions of relatively imminent and significant damages to the economy has resulted in treaties such as the Paris Agreement. Under this, 195 countries agreed to limit the long-term global temperature rise below 2°C above pre-industrial levels. Thereby, it has led to a change in the narrative concerning the mitigation engagement from *whether* to *how*. The issue of financing the net-zero transition, however, still poses obvious economic and social obstacles. Most economic models of climate change produce optimal policy in the form of carbon pricing (e.g. Golosov et al., 2014; Barrage, 2020). This optimal policy is typically proposed to be implemented via carbon taxes or permits, however, due to political reasons, the introduction of carbon taxes remains rather slow.

According to Baranzini and Carattini (2017), public opposition against carbon taxes often stems from the scepticism regarding the issues like distributional impacts on poor households or effects on employment and competitiveness. Moreover, the authors point to the general disbelief in environmental effectiveness of carbon taxes, despite the theoretical economic rationale. Such opposition manifests especially when carbon tax policy plans are announced to the public. In practice, a vivid example of a public disapproval took place in 2016 and 2018 in the State of Washington. The citizens participating in the referendums rejected the proposal for a tax of $15 per ton of CO_2. Given the closeness of the 2050 goal declared in the Paris Agreement, effective climate policy appears to require measures that are politically feasible.

In this paper, we examine a novel approach to climate policy, which nevertheless relies on standard instruments. We consider pension tax relief as a potential vehicle to conduct climate policy. Namely, could a policy of requiring pension funds to invest in "green projects" in exchange for their tax relief advantages constitute a reasonable alternative to carbon taxes? The primary argument favouring our approach concerns its potential political feasibility, especially for countries that already grant some form of tax relief on pension savings, such as e.g. the United Kingdom, the United States or Belgium. Utilising an existing policy might prove more acceptable to the public as it does not explicitly involve introducing new taxes.

In practice, several countries have, to some extent, incorporated pensions into broader climate policy. However, rather than using them as an actual tool, policymakers rely on the interplay between carbon tax revenue and its subsequent recycling. For instance, Norway created a public pension fund continually financed by income from oil drilling licences and carbon taxes (Sumner et al., 2009), whereas Germany reallocated the bulk of the eco-tax receipts to the existing public pension system (Weidner, 2008).

In terms of studies, perhaps the ones closest conceptually to our paper are the ones of von Below et al. (2016) and Dam (2011), both of which utilise the Diamond (1965) model. Apart from these papers (which we discuss in more detail in section 2), however, economic modelling literature oriented on exploring fiscal alternatives or complements to carbon taxes remains largely silent on the notion of pensions as a possible element of climate policy.

To address this gap, we combine the well-studied tax relief or subsidy approach with a novel way of its application. We model existing tax relief on pensions savings to see if variation in its form or rate could be used as climate policy. Specifically, we model a Diamond-type dynamic overlapping generations economy to differentiate between workers and pensioners, and thereby to directly consider pension savings. We calculate the constrained optimum policy, subject to instruments available to policymaker. The policies that we evaluate include our proposed policy of tax relief on green pension investment, a standard policy of tax relief on entire pension investment, as well as a policy mix of the latter with carbon taxes.

Given an OLG model with an environmental externality, there are two sources of inefficiency: the incomplete markets problem of being unable to trade with unborn future generations, and the global environmental externality. Clearly, therefore, implementing the socially optimal solution requires two independent policy instruments such as pensions tax relief and a carbon tax. Finding the constrained optimum subject to being able to use only pensions tax relief inevitably cannot achieve this. Nevertheless, the green pension policy, in implementing this constrained optimum, might constitute a reasonable - and potentially politically acceptable - alternative to carbon taxes. Although it is associated with a temperature rise of 2.3°C (i.e. additional 0.3°C relative to the goal consistent with the Paris Agreement) in our calibration, such a policy yields a negligible welfare loss compared to the social optimum.

Above all, our paper's primary contribution is conceptual and relates to the growing "second-best" climate change economics literature (discussed in section 2). While we rely on a well-studied tax relief approach, we model and evaluate – to the best of our knowledge – a novel proposal involving pensions as a climate policy tool.

The rest of this report is structured as follows: the next section provides an overview of the literature oriented on alternative climate policy instruments; section 3 introduces the baseline model; section 4 discusses the optimisation process related to the social planner's solution and climate policy variants; section 5 reports the numerical simulations and evaluates the policies; and section 6 concludes.

2 Alternatives to carbon taxes in the literature

2.1 Pensions

The two studies which rely on the Diamond (1965) model and tackle the issue of pensions in the environmental context are by von Below et al. (2016) and Dam (2011). The former proposes a Pareto-improving deal to resolve the trade-off between the coexisting generations. In principle, current pensioners would not experience the benefits of costly mitigation efforts (financed e.g. by carbon taxes). Therefore, to make them indifferent, the younger generation shall compensate them via pension transfers. The young, in turn, inherently save less, although expect to ease the climate damages they will experience in the future. Thus, they become better off if the discounted value of their own prospective retirement returns exceeds the loss attributable to the *total* abatement costs. What stems from such a bargain is that the economy experiences a substantial increase in overall abatement, and a higher price of carbon becomes acceptable.

Dam (2011) relates to pensions only implicitly, through attention to retirement consumption. Nevertheless, the author suggests that the intergenerational coordination problem (resulting in overaccumulation of pollution) could be resolved through the security market and socially responsible investment funds which attach environmental quality to the firm's intrinsic value. The forward-looking character of the financial market then incentivises the young to reduce pollution (to sustain firm's value and, therefore, funds available for consumption after they retire) and to indirectly consider the impact on future generations, implying no corrective policy is necessary.

2.2 Subsidies and tax reliefs

Other, possibly more diverse in design or application, prevalent instruments are subsidies and tax reliefs. As Aghion et al. (2014) recognise, while discouraging dirty production, carbon taxes alone provide limited means to induce a swift development of clean technologies. For instance, innovation may initially focus around the efficiency of combustion (however important in the green transition, too), rather than on renewable solutions. Standard climate policy, according to the authors, should therefore be reinforced with government subsidies, such as green investment tax breaks. Hoel (2012) claims that a second-best policy involving green subsidies is justified when the existing price of carbon is set below its optimal level – one of the reasons being e.g. public opposition. Moreover, Acemoglu et al. (2012) and Golosov et al. (2014) argue for the vital role of subsidies, especially for the economy to endogenously direct resources toward green technology.

The tax relief approach is among the climate tools evaluated by Monasterolo and Raberto (2016). It is used to stimulate investment in renewables but is found to comparably depress the overall economic performance (hence, the authors advocate a green monetary policy). However, the fiscal policy still performs better than the business-as-usual scenario, providing a rationale for further consideration.

Kalkuhl et al. (2013) demonstrate a comprehensive welfare analysis of four second-best regimes involving a subsidy to renewable energy: "feed-in-tariff" where subsidies are financed by taxing the energy sector (fossil and nuclear alike); "carbon trust" where carbon tax on emissions is recycled in full toward renewables; "renewable energy subsidy" where the pure subsidy is financed by lump-sum taxes on households; and "temporary subsidy policy" where initial subsidies are gradually displaced by carbon taxes (advised particularly when optimal carbon taxes are not politically viable in the short-run). Relative to the first-best optimal carbon pricing scheme, the highest consumption losses are associated with the pure subsidy policy. On the other side of the spectrum is the carbon trust policy, with the feed-in-tariff marginally more costly. The temporary subsidy approach, in turn, can be the closest to the optimum; however, only for a shorter displacement window – the longer it takes to replace the subsidy with carbon taxes, the higher the welfare losses.

2.3 Role of financial market

Subsidies are also explored in relation to another area of interest (even if our paper tackles the issue only implicitly): the financial market. Renström et al. (2021) extend Dam's (2011) framework with socially responsible investors and develop a dynamic general equilibrium model where individuals can choose between a firm's shares and green government bonds. The firm's share value is determined chiefly by its production and its "cleanness rating". Therefore, the firm can decide to abate to avoid paying higher pollution premia to the investors - which the existing system of pollution tax and abatement subsidy should further incentivise. Ultimately, the authors find higher pollution taxes to decrease pollution successfully but at the cost of the economy's performance and individual consumption. On the other hand, increased subsidies still contribute to pollution mitigation (although less effectively) while improving the scale of the economy and consumption. Their results seem to suggest that a politically feasible complement to climate policy has a potential to exert indeed positive economic impacts – in socially responsible environments, at least.

A simpler fiscal policy in a similar setting is studied by Dam and Heijdra (2011). They assess the interaction between public abatement funded by lump-sum taxes and socially responsible private investment, however without any policy that would further incentivise such efforts. The key finding from the paper is that socially responsible investment partially offsets the positive impact of public mitigation due to the crowding-out effect.

Lastly, in the contemporarily important context of developing countries, Davin et al. (2023) examine the environmental impacts of debt relief combined with public abatement. Their OLG model considers a bilateral agreement between a high and low-income country where public debt reduction of the latter (channelled through the financial market) is financed by the richer one. Although the study does not reflect on the optimality of the solution, it suggests that environmental quality can indeed improve and – depending primarily on how mobile assets are – both countries can experience welfare enhancement as a result of the debt transfer.

Overall, the literature considers various policy designs which could aid the transition towards a net-zero economy. However, it appears that the economic modelling literature has not yet properly addressed the possibility of conducting a climate policy through pensions. Moreover, a clear gap remains with respect to the evaluation of the specific policy which would grant tax relief on green pension savings only.

3 The model

In the following section, we introduce the general model based on the overlapping generations framework developed by Diamond (1965). The economy is characterised by the simultaneous existence of two finitely-lived generations of people. These individuals are assumed to live for two periods. At each period t a new generation of workers enters the labour force, earns labour income and makes consumption-saving decisions. At the beginning of the subsequent period, the generation transforms into pensioners who live on capital income. Similarly, a generation of existing pensioners dies every period and exits the model.

Firms are perfectly competitive and pay labour and capital their marginal products. The model also considers the environment: there is an externality caused by production which damages utility flows. Such negative impacts can be reduced by private abatement spending.

Below we expose the baseline model's components, establish their elemental dynamics and study the general behaviour of the key variables. Later, in section 4.2 we introduce extensions which capture specific climate policies.

3.1 Environment

To model the externality, we firstly define how environmental quality, E_t, evolves over time. Later, in section 3.3, the variable is employed to the utility function of households. After John and Pecchenino (1994), one can interpret E_t in multiple ways, ranging from the quality and cleanliness of water to certain biodiversity measures. We interpret it as some measure of the inverse of the greenhouse gases concentration, an index of climate change-related performance or simply *climate* in general. Later, in the calibration section, we translate the changes in E_t to the rise of global average temperatures.

We do not consider any source of possible natural degradation or recovery and assume the environment only to change in line with net emissions. It worsens as a result of output production, Y_t, net of private abatement a_t. The latter may include all activities aimed at reducing the potential concentration of greenhouse gases, such as energy-efficiency enhancement or shift to alternative energy sources – aggregated simply by the notion of green projects or green investment. We adapt Davin et al. (2023) specification – which itself is an incarnation of the widely recognised design of John and Pecchenino (1994) – so that it serves a single-economy case with private mitigation. The evolution of environmental quality "stock" is therefore expressed as

$$E_{t+1} = E_t - \theta Y_t + \phi a_t, \tag{1}$$

where Y_t and a_t denote global output and abatement. The actual marginal effect that contemporaneous production and mitigation activities exert on the environment is captured respectively by the emission factor $\theta > 0$ and the efficiency of abatement factor $\phi > 0$. These parameters are assumed $\phi > \theta$ and constant[1].

The benefit of formulating E's evolution as a function of net emissions and the current state of the environment is that environmental quality "memorises" the past accumulation of those emissions. This way, it can quite realistically reflect the fact of storing the greenhouse gases in the atmosphere (i.e. the gases tend to remain there for a relatively long time), consistent with e.g. Dietz et al. (2020). There is no theoretical upper bound related to the value of E, however, we assume, via preferences that are discussed in Section 3.3, that $E_t = 0$ would be associated with a climate catastrophe or human extinction.

Moreover, we can see that, holding all else constant, an additional unit of abatement is always beneficial to the environment. It is also equally beneficial across its all levels, as given by the constant returns to abatement. Likewise, output exerts a constant, negative marginal effect on environmental quality.

3.2 Firms

We assume a simple Cobb-Douglas specification for the supply side of the economy. Firms produce output using labour and capital which are inelastically provided by workers and pensioners respectively:

$$Y_t = {L_t}^{1-\alpha} K_t^{\alpha}, \qquad L, K > 0 \tag{2}$$

where α symbolises capital's share in production or, more generally, output's elasticity with respect to this input.

[1]The $\phi > \theta$ assumption seems reasonable - to fight climate change, we expect the mitigation efforts to be at least as efficient as the damages per unit of output - but follows mainly from how equation (1) is designed. To illustrate, let us imagine a hypothetical, extreme case where all output was dedicated to abatement spending, so that $a_t = Y_t$. Then, for the environmental quality to improve (i.e. $E_{t+1} > E_t$), we would still need the abatement efficiency factor to be higher than the emission factor. This can be seen by rewriting equation (1) and assuming $a_t = Y_t$: $E_{t+1} - E_t = (\phi–\theta)Y_t$. Then, for the LHS to be positive the $(\phi–\theta)$ subtraction also needs to be positive. This is ensured when $\phi > \theta$.

Profit-maximising behaviour requires firms to hire labour and capital up to the point where the marginal products of these inputs equal their prices. Hence, in the competitive economy, firms earn zero profits and pay the workers and pensioners the equivalent of their marginal products. Wage and rate of return on capital are therefore

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)\frac{K^{\alpha}}{L^{\alpha}} = (1-\alpha)\frac{Y_t}{L_t} \tag{3}$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha L^{1-\alpha}K^{\alpha-1} = \alpha\frac{Y_t}{K_t}. \tag{4}$$

3.3 Preferences and budget constraints

Each period t, a homogeneous representative consumer who enters the workforce chooses – based on the current state of the economy and environment – a mix of consumption, investment and abatement which maximises their lifetime utility. The latter – which is additively separable in its arguments – is defined similarly to Davin et al. (2023) as

$$U_t^i = lnC_t + \beta lnC_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1}. \tag{5}$$

Households have logarithmic preferences and β denotes the discount factor[2]. Workers care about consumption, C, in both periods of their life, but also consider the environmental quality, E, they experience, subject to the environmental sensitivity factor ϵ.

Individuals are assumed to follow a price-taking behaviour: there is a unit-mass of identical, infinitesimal households that do not internalise the possible economy-wide implications of their decisions on prices. We normalise the size of each generation to 1 and assume no growth in population. Workers receive a gross wage, W_t, which is used for current consumption and savings. Savings can be allocated in the form of either productive brown investment or abatement which will improve the environmental quality to be experienced after retiring.

Pensioners no longer earn W and consume all the proceeds from the income invested in period t while being workers (we do not consider any bequest motive so pensioners fully utilise their available budget). The said proceeds - or *pension* payments - are basically the return, r_{t+1}, realised on renting the accumulated capital to firms. The general case, therefore, results in the following budget constraints:

$$W_t = C_t + I_t + a_t \tag{6}$$

$$r_{t+1}K_{t+1} = C_{t+1}. \tag{7}$$

[2]Which formally is defined as $\frac{1}{1+\rho}$, where ρ symbolises the individual discount rate.

As implied, workers face a trade-off between saving in a way that allows consumption at old age (I_t) and a so-called "green investment" (a_t) conducive to the state of the environment. Effectively, we consider two types of assets in which workers can invest their savings: physical "brown" capital and "green projects" concerning the totality of intangible assets[3] oriented on financing emission-reducing endeavours. Investment in brown capital is more desirable in monetary terms (generates return r) but harms the environment, whereas green investment is not financially rewarding ($r = 0$) but instead improves environmental quality[4].

In regards to the dynamics of capital stock, the typical law of motion for capital applies:

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{8}$$

The above equation simply states that the stock of capital in the following period is determined by a sum of the current capital stock (subject to depreciation) and new investment. Capital is assumed to fully depreciate, what constitutes a reasonable assumption, considering that one t reflects half the lifetime of a generation. Thus, δ is set to 1 and new capital stock at $t + 1$ is determined solely by the investment outlays from the preceding period. Therefore, for the remainder of this paper, we skip the $1 - \delta$ parameter in the specifications and calculations.

4 Optimisation

In this section, we analyse the theoretical solutions to the optimisation problems faced by the agents in the model. This will serve as a groundwork for the numerical simulations discussed in section 5. Below we synthesise the baseline model - in the absence of any fiscal policy - specified in the previous section. From the perspective of an individual worker, the intertemporal relationship between the variables is primarily founded on the following equations:

$$W_t = C_t + I_t + a_t \tag{6}$$

$$r_{t+1}K_{t+1} = C_{t+1} \tag{7}$$

$$K_{t+1} = I_t \tag{8}$$

$$E_{t+1} = E_t - \theta Y_t + \phi a_t \tag{1}$$

$$Y_t = K_t^{\alpha} \tag{2}$$

In the model described above, a dynamic competitive equilibrium is characterised by a sequence of $\{K_t, E_t, C_t, I_t, a_t\}_{t=0}^{\infty}$ and a price path $\{r_t, W_t\}_{t=0}^{\infty}$ such that, for any given K_0 and E_0 , utility is maximised subject to the resource constraints, firms optimise (zero) profits and markets clear. By optimising the use of inputs, firms intrinsically dictate the equilibrium prices of inputs or income rates for

[3]Theoretically, they might be eventually realised in a physical form. Green capital formation, however, is not considered in this model.

[4]We assume the workers believe their abatement decisions indeed matter. In section 4.2, we discuss why it need not be the case.

households. Hence, firms' optimising behaviour can be implicitly evidenced by plugging the respective expressions for marginal products into wage and return identities. The interaction of the optimising behaviours then provides the general equilibrium and ensures the clearing condition.

The overall consumption-saving problem in this section is fourfold. Firstly, we characterise the solution of a central authority who is in the capacity to decide on choices on behalf of the households. Then, we turn to decentralised optimisation and solve the updated problem involving three different climate policies.

4.1 Social planner's solution

First of all, we characterise the social planner's solution, that is we maximise social welfare subject to the aggregate resource constraints, ignoring the policy levers that may implement such an allocation. The optimal plan we construct – due to the forward-looking character of the planner and ability to distribute resources in a manner not available to the market – shall also be efficient in the Pareto sense, in line with Blanchard and Fisher (1989). Therefore, the social planner overcomes the possibility of dynamic inefficiency often arising in the Diamond model and provides an idealised welfare benchmark for climate policies explored later.

The social planner shall have the entire output, Y_t, produced in the economy at their disposal, to be redistributed between consumption for both generations alive at t, investment in dirty capital (used to produce output in the following period) and abatement. The general form of the aggregate resource constraint, therefore, is written as[5]

$$Y_t = C_t^{workers} + C_t^{pensioners} + I_t + a_t. \tag{9}$$

Apart from the budget, the planner operates subject to the same laws as the rest of the economy. Namely, must follow the evolution of environmental quality (1) and the law of motion for capital (8). As of the latter, by combining it with (9) we can rewrite it in budget constraint terms, i.e.

$$K_{t+1} = Y_t - C_t - C_t' - a_t \tag{10}$$

to obtain an economy-wide capital stock accumulation identity available to the social planner, which intrinsically bounds the consumption-saving decisions. We implicitly consider brown investment, I_t, a control variable, although expressing it as a function of the resource constraint simplifies the calculations without any loss of generality. Then, by expressing Y_t as a function of capital, we eliminate the remaining superfluous variable to settle with

$$K_{t+1} = K_t^{\alpha} - C_t - C_t' - a_t \tag{11}$$

[5]Note that for clarity we add the superscripts referring to the specific generations. From this point onward, however, the notation will take the form of C_t applicable to consumption of workers and C_t' denoting consumption of pensioners.

and

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t. \tag{12}$$

A feasible allocation denotes any sequence of choices for consumption and abatement $\{C_t, C'_t, a_t\}_{t=0}^{\infty}$ which satisfies the aggregate resource constraint (11). We are, however, interested in a choice rule which – for any potential value of the states – will maximise social welfare and, therefore, consider the aggregate utility. The social planner aims to maximise the joint welfare of all generations to come. This means that at each t the planner considers – subject to the social discount factor – the respective, contemporaneous utility of both generations alive at the same time. Thus, although the planner discounts future households' welfare, the concurrent generations are treated equally, without attaching weight to any specific age group. Imposing $C'_t = \alpha K_t^{\alpha}$ (such that pensioners simply consume their income), we consider the following social welfare function:

$$U_t^{social} = \sum_{s=0}^{\infty} \beta^s (lnC_{t+s} + ln(\alpha K_{t+s}^{\alpha}) + 2\epsilon lnE_{t+s}) \tag{13}$$

s.t. (11) & (12). The optimisation is solved through the dynamic programming method. By assuming that the control variables are chosen optimally, we can then write the value function in terms of the states to obtain the following Bellman equation:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = K_t^{\alpha} - C_t - \alpha K_t^{\alpha} - a_t \tag{15}$$

and

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t. \tag{12}$$

Based on the above, the existence and optimal characterisation of the general equilibrium can then be described by the first-order conditions taken with respect to the control variables C_t and a_t, and envelope theorem conditions obtained with respect to the state variables K_t and E_t. They, together with the description of the whole process, are found in Appendix 8.1

Knowing the initial values of the states, the 4-dimensional system of dynamical equations in four unknowns enables to numerically simulate the model forward and produce complete optimal paths. The motion of capital is denoted by an implicit equation (16); the evolution of consumption is based on the "Euler" identity (17); environmental stock is simulated forward using (18); whereas the last equation (19) allows to infer the consistent choice of abatement spending:

$$\left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right]^{-1} = \frac{1}{\beta C_t} - \frac{2\epsilon\phi}{E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1})} \tag{16}$$

$$C_{t+1} = \left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)^{-1}\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right] \tag{17}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1}) \tag{18}$$

$$a_t = (1-\alpha)K_t^{\alpha} - C_t - K_{t+1} \tag{19}$$

We can thus see that consumption allocation is dependent on the emission and abatement efficiency factor: the greater the $\frac{\theta}{\phi}$ ratio (i.e. the "more difficult" the abatement undertaking) the lower consumption at $t+1$. Put differently, a higher marginal externality shall lead to lower consumption in the next period. This is consistent with the fact that greater future consumption requires greater investment outlays, which in turn harms the environmental quality. The social planner, however, allocates the optimal consumption having incorporated the full societal impacts related to this choice, i.e. including environmental consequences.

It is evidenced at least since Zhang (1999) that non-trivial dynamics are likely to occur in environmental-growth models. Nonetheless, in the case of this relatively simple – numerically-wise – model, we can presume saddle-path stability and convergence to a steady state. This can be safely assumed to be guaranteed by infinitely negative utility as $E \longrightarrow 0$. The system (16)-(19) is solved by a forward shooting algorithm so that we tend towards the steady state given by (20)-(23). The steady state has analytic solutions given as ("star" signs denote the steady state values):

$$K^* = \left[\frac{2\alpha\beta(1-\alpha-\frac{\theta}{\phi})}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}} \tag{20}$$

$$a^* = \frac{\theta}{\phi}(K^*)^{\alpha} \tag{21}$$

$$C^* = (1-\alpha)(K^*)^{\alpha} - K^* - a^* \tag{22}$$

$$E^* = \left(\frac{\beta}{1-\beta}\right)2\epsilon\phi C^*. \tag{23}$$

Our model features no ambiguity concerning the direction in which the social discount rate influences the steady state values. Therefore, a theoretical change in the discount rate does not trigger the opposing channels prevalent in some OLG models (such as Gutiérrez (2008)) in which, on the one hand, a lower discount rate translates to higher levels of capital stock provided to the future generations, and on the other, through capital's link to externality, suggests that future provision of welfare inherently reduces capital stock. Here, owing to the ability to mitigate, a lower discount rate always results in a higher level of steady state capital stock and environmental quality alike, as follows from the analytical solutions. However, the same swings in the discount rate would be associated with unequal long-run marginal effects. It can be shown by isolating the $\left[\frac{2\alpha\beta}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}}$ and $\left(\frac{\beta}{1-\beta}\right)$ factors from (20) and (23) respectively, that environmental quality is more sensitive to such changes than the capital stock.

Basic steady state impacts of some of the remaining parameters or variables summarise quite logically, too, if we dissect the system and analyse holding all else constant. Firstly, the greater the ratio of the emission factor θ to the efficiency of abatement ϕ, the lower the steady state stock of capital. Conversely, one can observe that abatement activity needs higher (lower) levels of spending if the said ratio is greater (smaller). At the same time, (21) highlights that, *ceteris paribus*, mitigation efforts are stronger for higher levels of the steady state capital stock. On the one hand, it reveals that higher output allows more funds to be dedicated to green projects. On the other, it suggests that greater production requires more compensation for environmental damages. Continually, consumption available to workers inevitably depends on the aggregate capital stock in the economy (in particular through steady state wages), although the abatement's opposing channel partially outweighs its amount. Lastly, the state of the environment is positively associated with the concurrent generations' sensitivity factor ϵ: it is clear that higher sensitivity requires more emphasis on climate maintenance.

4.2 General individual problem

The decentralised economy implicitly involves incomplete markets (i.e. current generation cannot trade with their "grandchildren"). Therefore, decentralisation of the socially optimal solution needs to include corrective measures manifested through a fiscal policy. The individual general problem is thus as follows:

$$U_t^i = lnC_t + \beta lnC'_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1} \tag{24}$$

$$C_t = T_t + (1-\tau_t^p)W_t - \tau_t^c(\theta K_t^\alpha - \phi a_t) - (1-\tau_t^I)I_t - (1-\tau_t^a)a_t \tag{25}$$

$$C'_{t+1} = r_{t+1}I_t \tag{26}$$

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t \tag{12}$$

where transfers (taken as given) are:

$$T_t = \tau_t^p(1-\alpha)K_t^\alpha + \tau_t^c(\theta K_t^\alpha - \phi a_t) - \tau_t^I I_t - \tau_t^a a_t \tag{27}$$

In the general case, therefore, individuals may face a combination of labour income taxes and carbon taxes. The latter, τ_t^c, is set in proportion to net emissions.

Thus, the amount of the tax levied increases with production and is reduced in line with abatement undertaken during the same period. Carbon tax is refunded as part of lump sum transfers, T_t.

The labour tax in this setting is an instrument of the general pension policy[6]. The tax is reallocated as a tax relief (and, effectively, as a subsidy) to brown and green investment. Any residual balance of the tax revenue which was not used directly for the relief would be refunded through transfers to workers (pensioners are omitted for simplicity to reduce recursion in the model); this ensures that the policy is revenue-neutral to the fiscal authorities.

Regarding the evolution of the environmental quality (12), however, it might be argued that atomised individuals do not believe their mitigation efforts matter. Hence, as far as their optimisation is concerned, no τ_t would encourage it. In this regard, from the perspective of an individual worker, the actual expression for the evolution of environmental quality could resemble $E_{t+1} = E_t + \theta K_t^{\alpha} + \phi \int_0^n a_{i,t}\, di$. We do not consider such a formulation in this paper, instead relying on assumptions which ensure abatement spending's sensibility is acknowledged.

We assume the existence of a financial intermediary (i.e. pension funds) that rewards private investment in abatement to the extent that it is valued in the aggregate. The financial sector allows individuals to make abatement decisions assuming that everyone else makes the same abatement decision, and thus that their individual abatement decision matters for the aggregate environmental outcome.

Alternatively, we can think about their preferences in (24) in terms of the warm-glow effect, similar to the specification used by Dam (2011) in his Diamond model with socially responsible investors. According to this, workers would derive satisfaction from the very fact of doing something considered ecological, rather than from affecting the environmental quality per se.

Lastly, we can simply refer to the seminal work of John and Pecchenino (1994) who also rely on the OLG specification with private mitigation and environmental quality. They optimise individual behaviour without sharing our concerns and thereby assume that workers believe their individual abatement matters. This appears in line with Fodha and Seegmuller (2012) who assume strictly positive private abatement in their model. They note that positive private abatement is supported by empirical evidence. Therefore, regardless of the specific assumption

[6] At this point, it appears worth reiterating that we refrain from modelling distinct pension market components. Instead, we benefit from the inherent design of the overlapping generations model. For instance, Blake (2006) states that a fully funded pension system formulated under this framework effectively results in an identical outcome as in the specification with no formal system whatsoever, i.e. consisting of private savings only. Owing to this, the pension tax relief policy can be implemented simply by introducing the policy variable τ^p denoting the pension tax.

(financial intermediary or the warm-glow effect), workers in our model believe their individual mitigation spending can affect the state of the environment so that (12) holds.

The individual optimisation process is described in more detail in Appendix 8.2. We impose $a_t \geq 0$ constraint[7] and obtain the following system:

$$a_t = \left[(1-\alpha)K_t^{\alpha} - \left(\frac{1-\tau_t^I+\beta}{1-\tau_t^I}\right)\frac{1}{\epsilon\beta\phi}\Big(1-\tau_t^a-\tau_t^c\phi\Big)\Big(E_t - \theta K_t^{\alpha}\Big)\right]\frac{\epsilon\beta\Big(1-\tau_t^I\Big)}{\epsilon\beta\Big(1-\tau_t^I\Big)+\Big(1-\tau_t^I+\beta\Big)\Big(1-\tau_t^a-\tau_t^c\phi\Big)} \tag{28}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\Big(1-\tau_t^a-\tau_t^c\phi\Big)\Big(E_t - \theta K_t^{\alpha} + \phi a_t\Big) \tag{29}$$

$$I_t = \frac{\beta}{1-\tau_t^I}C_t \tag{30}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t \tag{12}$$

Equation (29) logically uncovers a negative relationship between the taxes level and consumption choice. Similarly, according to (28), higher tax rates would be associated with an increased abatement, *ceteris paribus.*

With two control variables in the social planner's solution and two sources of inefficiencies in the decentralised model (environmental externality and incomplete markets), we need two levers to fully replicate the social optimum: a labour income (pension) tax and a carbon tax. Therefore, the social planner's solution can be implemented when $\tau_t^c > 0$ & $\tau_t^p = \tau_t^I = \tau_t^a \equiv \tau_t^p$. By setting the tax rates optimally (see Appendix 8.2), the evolution of the (28)-(30) & (12) system can match the evolution of the social planner's solution.

[7] If the equation (28) yields a negative number, we impose $a_t = 0$ and split aggregate output over C_t, C_t' and I_t, s.t. $C_t' = \alpha K_t^{\alpha}$ and $C_t + I_t = (1-\alpha)K_t^{\alpha}$ and $I_t = \frac{\beta C_t}{1-\tau_t^I}$, such that $C_t = \frac{1-\tau_t^I}{1-\tau_t^I+\beta}(1-\alpha)K_t^{\alpha}$.

4.3 Green pension policy

The green pension policy discards the carbon tax and involves only the income tax. Here, however, it is used to subsidise only green investment. Individual consumption-saving decisions still follow the optimised system specified in section 4.2 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$, $\tau_t^I = 0$ & $\tau_t^p = \tau_t^a$. Accordingly, transfers become:

$$T_t = \tau_t^p(1-\alpha)K_t^\alpha - \tau_t^p a_t \tag{31}$$

Recall that we cannot replicate the social planner's solution with only a single policy variable. Hence, whereas households optimise their behaviour *subject to* the tax rate, we are interested in obtaining the solution that the planner would impose if they were subject to the constraint of only using this policy lever. The planner will take into account how workers form their decisions and, based on this, choose a sequence of tax rates τ_t^p to maximise social welfare. The planner has a single policy lever, τ_t^p, which by equations (28)-(30) uniquely determines consumption, abatement and investment. Therefore, in stating the dynamic programming problem, we can express using which control variable is most convenient. In the following, we use consumption. The planning problem is:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^\alpha) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = I_t = \beta C_t \tag{32}$$

and

$$E_{t+1} = E_t + \phi\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^\alpha - \phi(1+\beta)C_t. \tag{33}$$

Full details can be found in Appendix 8.3, but in principle, we rewrite the decentralised system to express it in terms of the states and only a single choice variable, C_t. Thus, rather than tax rates, the planner can equivalently construct the full optimal plan based on consumption choices[8].

4.4 Standard pension policy

The last of the evaluated policies in principle resembles the pension policy that already happens in practice, i.e. tax relief is granted on all pension investment, regardless of its "dirtiness". Here, however, we optimise the policy such that it intentionally serves the climate goals, too.

As before, we rely on the system given by equations (28), (29), (30) and (12), now with $\tau_t^c = 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$. Transfers are given by:

$$T_t = \tau_t^p(1-\alpha)K_t^\alpha - \tau_t^p I_t - \tau_t^p a_t. \tag{34}$$

[8] As described in Appendix 8.3, optimal tax rates can then be inferred based on those choices.

Following the procedure from 4.3, the social planner aims to maximise social welfare, subject to the individual constraints. In this case, all savings are subsidised relative to consumption, but the planner cannot influence the relative level of brown against green investment. They construct the plan using a series of equivalent C_t choices to maximise:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = \frac{1}{1+\epsilon}\left[\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^{\alpha} + \frac{E_t}{\phi} - C_t\right] \tag{35}$$

and

$$E_{t+1} = \frac{1}{1+\epsilon}\left[E_t + \epsilon\phi\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^{\alpha} - \epsilon\phi C_t\right] \tag{36}$$

where full details can be found in Appendix 8.4.

5 Numerical simulations

5.1 Calibration

The model is calibrated primarily such that the environment at its steady state - associated with the social planner's solution - is consistent with the 2°C above the pre-industrial temperature target. The parameter values that ensure this condition are found in the table below.

β	θ	ϕ	ϵ	α
0.67	2.53	50.6	0.33	0.40

Table 1: *Parameters*

The procedure which we follow to establish the stated values is as follows. Firstly, we declare the initial level of capital stock characterising the economy. This is achieved by solving for the steady state of the optimised model with no environmental problem considered. Full details are in Appendix 8.5 and the equation which is of our interest here is given by

$$K_1 = \left(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta}\right)^{\frac{1}{1-\alpha}} \tag{37}$$

Therefore, equation (37) specifies the initial stock of capital, K_1, in our baseline model. The initial value of environmental quality, in turn, is assumed to reflect the current global warming of 1°C since the pre-industrial levels. To capture this in terms of units, we further arbitrarily assume that human extinction is associated with 10°C warming: for this level of $E_t = 0$, the agent's utility would tend to $-\infty$. Hence, the initial situation can be expressed as $E_1 = E^{max} - 1 = 9$. At the same time, we can thus define the assumed goal of 2°C which shall reflect the steady state of the social planner's solution and which is given simply by $E^* = E^{max} - 2 = 8$.

Having declared the initial states and assuming $\beta = 0.67$ (which shall reflect the choice of a discount rate of 2% and a generation's lifetime of 20 years), we can specify consistent values of the remaining parameters. Firstly, again referring to the model with no externality, we solve for the value of α. Essentially, similarly to the model variant specified in section 4.3 (however, without environment and abatement), we consider the decentralised solution with a pension tax optimised by the social planner (such that the planner corrects the incomplete markets inefficiency): α is calibrated to have an optimal labour tax, refunded on savings, of 10%[9].

Then, we note that estimates of the "carbon budget" consistent with keeping temperatures below a 2°C rise above pre-industrial levels, are typically of the order of 20 years of current emissions (see: MCC Berlin (n.d.)). Given our model has time periods of this length, we require $E^* - E_1 = -\theta K_1^{\alpha}$ (since this equates the social optimum environmental steady state with unabated production over the next 20 years). Combining equation (12) with (37) gives:

$$\begin{aligned} E^* - E_1 &= -\theta K_1^{\alpha} \\ 1 &= -\theta \left[\left(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta} \right)^{\frac{1}{1-\alpha}} \right]^{\alpha} \end{aligned}$$

through which we obtain θ. Additionally, let us suppose that in the steady state we need to spend 5% of national output on green investment, so that:

$$\frac{\theta}{\phi} = \frac{a^*}{(K^*)^{\alpha}} = 0.05$$

which yields consistent value of ϕ. Then, using equation (23) which specifies the steady state level of environmental quality in the social planner's solution, we can solve for optimal ϵ.

5.2 Social planner's optimal plan

Below, we deterministically construct an optimal, welfare-maximising plan and thereby declare a benchmark solution for the evaluation of the chosen climate policies. We use the system specified in equations (16)-(19) and rely on the forward shooting algorithm. By adjusting the choice of C_1, we can simulate - given the assumed initial levels of the state of the economy and environment - the entire model until the steady state is reached.

[9] This is targeted based on UK tax system, in which a significant proportion of labour income is taxed at 40%, but this is refunded on savings. These savings then generate retirement incomes which attract taxes at 20%. Aggregated over whole population, this may equate to a relative tax on working age incomes of 10%.

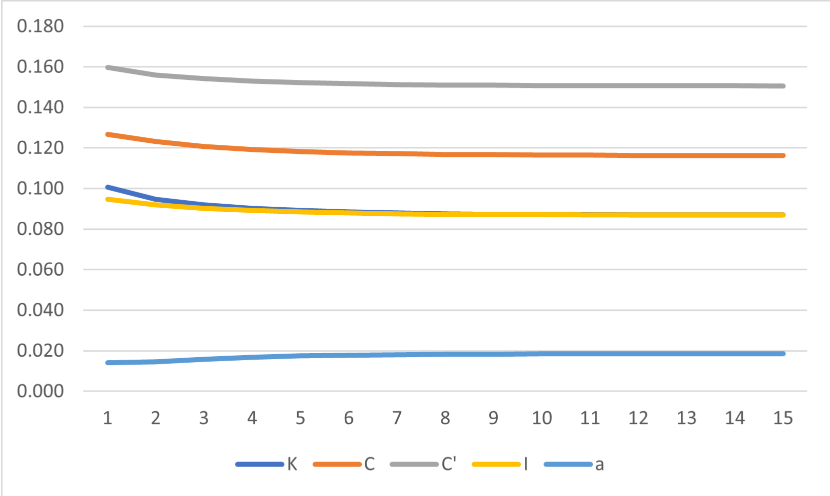

Figure 1: *Social planner's solution: paths of capital (K), worker consumption (C), pensioner consumption (C'), investment (I) and abatement (a) over time (t)*

In order to reach and keep the 2°C temperature rise target, the economy experiences relatively small but negative adjustments in the variables of interest (K falls by 14%, C by 8%, C' by 6%, and I by 8%), whereas abatement increases by as much as 32%[10]. The latter suggests that strong green action can be achieved without a proportionally large sacrifice to the economy and consumption. Regarding the overall evolution of the system, we can notice smooth and steady adjustment of the variables over time until they reach the steady state and become constant. A similar observation can be made with respect to the evolution of global temperature. Starting from the current 1°C rise, it takes roughly 11 periods to reach the assumed goal optimally and stabilise.

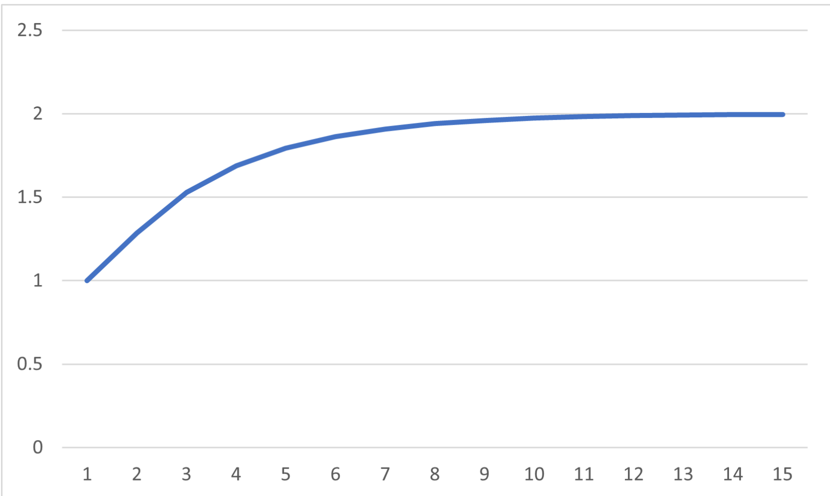

Figure 2: *Socially optimal temperature rise [°C] over time*

[10]Note that we do not show the initial period $t = 0$ associated with the no-externality steady state, in which abatement is still nonexistent.

5.3 Policy evaluation

Of the three policies specified in sections 4.2 - 4.4, only the one involving both the carbon tax and the pension tax can precisely replicate the social planner's solution. Hence, under this policy, the economy experiences the same adjustments as those shown in 5.2. Therefore, the two-taxes policy will constitute a benchmark for the remaining policies analysed in this section.

We begin the analysis by looking at paths of investment (see Figure 3). The relative levels are not surprising. Firstly, the green pension policy does not subsidise brown investment and therefore consistently features its lowest level. On the other hand, investment under the standard pension policy significantly exceeds the social optimum (by 49% in steady state). Moreover, we can notice the inverted U-shaped path of investment under the standard pension policy. This is a result of additional tax redistribution (due to the imposed $a_t \geq 0$ constraint) which further inflated brown investment[11].

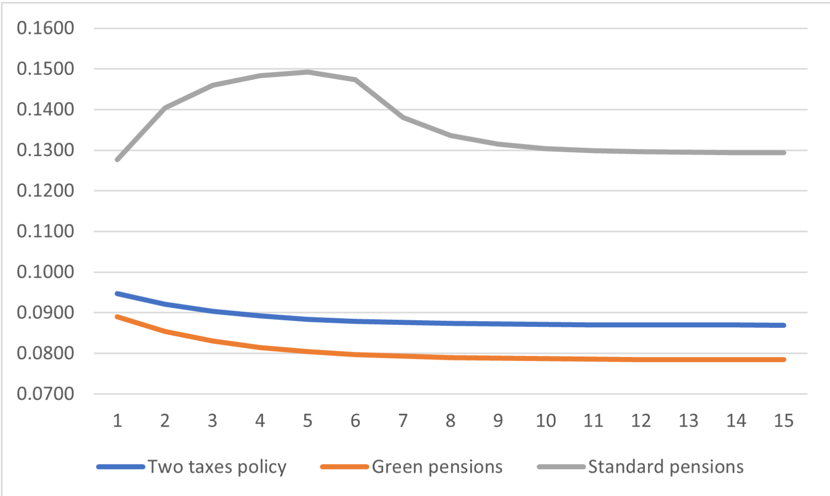

Figure 3: *Investment over time*

In terms of worker consumption, in Figure 4, we observe that initially the highest levels are associated with the green pension policy: finding investment spending not supported, individuals attach more value to current consumption. However, it gradually decreases (due to sub-optimal brown investment and, hence, output and wages "inherited" by consecutive generations) and equalises with the social optimum by the time the steady state is reached. Under the standard pension policy, initially, workers take advantage of the tax relief and prefer brown investment over consumption (hence substantial difference to the other policies). Nonetheless, consecutive generations of workers benefit from the "inherited" wages and begin to afford more consumption. The initial inverted U-shape again stems from

[11]This will become clearer once we analyse the paths of abatement. Essentially, the standard pension policy yields abatement spending non-optimal during the initial 5 periods. The entirety of the labour tax is therefore reallocated to subsidise brown investment.

the subsidy boost due to the $a_t \geq 0$ constraint. We notice that from $t = 3$, consumption surpasses the consumption levels related to the other policies. Following the halt of the extra redistribution (and positive, sizable, abatement spending commencing) at $t = 6$, however, consumption returns to significantly lower levels.

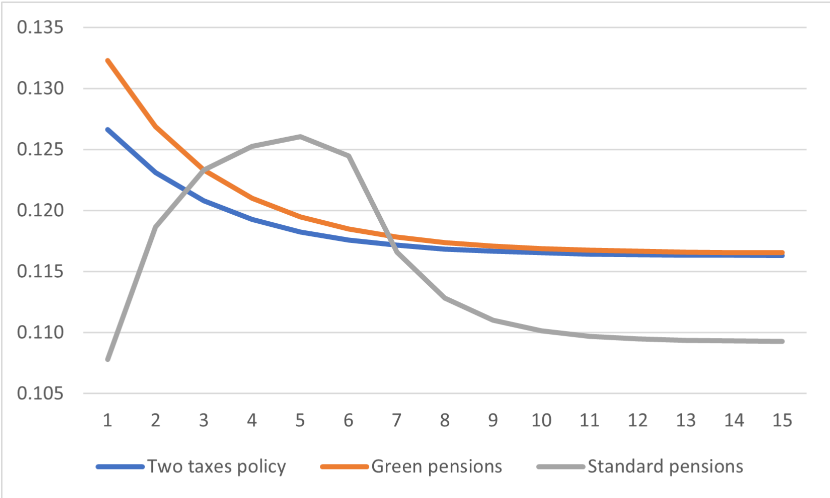

Figure 4: *Worker consumption over time*

Turning to pensioners, their consumption paths in all cases begin from the same level, which arises from the fact that the initial capital stock is not influenced by individuals in the model (i.e. pensioners at $t = 1$ "inherit" the existing capital stock and associated return on capital, irrespective of the policy). Unsurprisingly, the standard pension policy features consistently highest pensioner consumption (see Figure 5). This is a result of the higher investment, as discussed earlier. Conversely, the green pension policy - due to the lowest levels of productive investment - noticeably reduces consumption of pensioners.

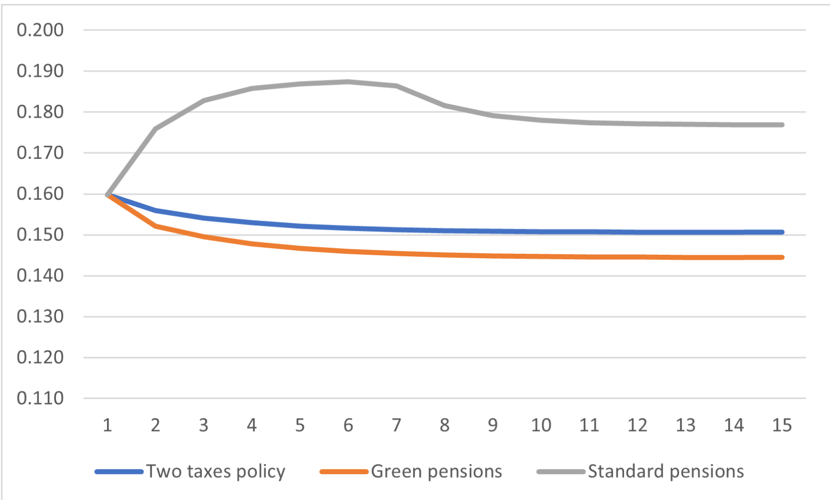

Figure 5: *Pensioner consumption over time*

Green pension policy, despite the subsidy, exhibits slightly lower abatement over the entire path, relative to the optimum (see Figure 6). We can look at this from three angles. Firstly, lower brown investment resulting from the policy translates to lower emissions. Thus, workers might be less pressed to abate, instead improving their consumption. Secondly, the policy does not feature the carbon tax, which would otherwise further incentivise abatement. Thirdly, we hypothesise (perhaps counterintuitively) that green savings are comparatively lower *because of* the sub-optimal brown capital formation. Namely, the green pensions subsidy appears to facilitate consumption and green investment at the cost of output production. The socially optimal solution, however, maintains a higher degree of brown investment (despite its negative environmental consequences) which allows to effectively dedicate more funds to future abatement.

Conversely, abatement under the standard pension policy is not incentivised sufficiently (i.e. income tax is split into two forms of investment). Thus, prior to $t = 6$, workers do not consider mitigation spending worthwhile and refrain from it completely, realising higher next-period marginal utility from consumption. After $t = 6$, when environmental quality reaches an "unacceptable" threshold, we notice a sharp increase in green investment to outweigh past negligence.

Figure 6: *Abatement over time*

Brown and green investment discussed above can be summarised by their impacts on climate. As depicted in Figure 7, whereas the optimal policy stabilises temperature at 2°C, the green pension policy produces additional 0.3°C (while abatement is lower than optimal, emission-inducing output production is lower, too. Hence, the difference in environmental performance is not striking). The standard pension policy, however, results in a global temperature growth reaching 7.8°C. Apart from the arguments discussed during the analysis of abatement paths (i.e. mitigation begins to take place too late), the decisive factor contributing to such an extreme rise is brown capital overaccumulation.

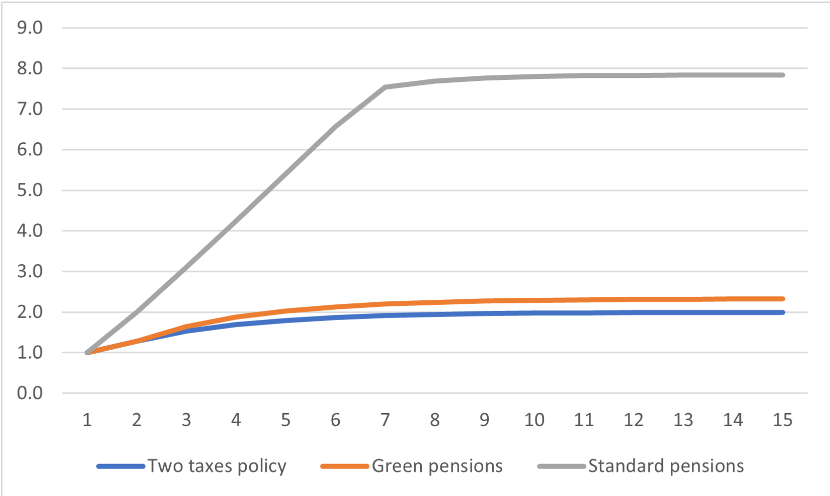

Figure 7: *Temperature rise [°C] over time*

Crucially, we are also interested in how the products of the policies translate to social welfare. Because the pension policies are not capable of fully tracking the social optimum, we might expect inevitably some degree of welfare loss, relative to the policy involving pensions and carbon taxes alike. Nonetheless, despite the noticeable differences in variables' evolution pointed out earlier, the policies deliver solutions of more comparable welfare effects. Because the social welfare measure reflects an infinite time-horizon of the aggregate utility, discounting diminishes the influence of the relative differences. Specifically, the overall social welfare loss attributable to conducting the climate policy through green pensions lays in negligible 0.1% regions. On the other hand, the standard pension policy results in a welfare loss of approximately 5%.

To conclude, we see a clear improvement related to the green pension policy, relative to the standard pension policy. Not only does it produce a nearly optimal outcome in social welfare terms, but appears to serve as a useful tool of climate policy. While, admittedly, the green pension policy produces additional 0.3°C on top of the assumed 2°C goal, it appears as a sensible - potentially socially acceptable - alternative to politically infeasible policies of carbon taxes.

6 Conclusion

This paper has argued that the introduction of an optimal carbon tax might face political constraints: the public might be reluctant to accept an announcement of a new tax. To address this, we propose a novel approach to climate policy, such that it relies on already existing taxes. Specifically, we develop and evaluate a model of conducting the climate policy through pensions. While typically policy-makers grant tax relief on pension savings irrespective of their potential impact on climate, we suggest that the relief could be granted only on "green" savings (which would be used for emission abatement projects).

To model our green pension policy, we rely on the Diamond-type overlapping generations framework. We define the tax relief such that a labour income tax is reallocated as a subsidy to green investment. We optimise the model and compare it with a standard pension policy where tax relief is granted on all pension savings, and with a policy which involves a combination of both the standard pension policy and carbon taxes.

We assume the optimal outcome to reflect the 2°C rise in global average temperature and evidence that the socially optimal policy can be conducted only with the aid of carbon taxes. While our green pension policy produces additional 0.3°C (i.e. totalling 2.3°C), we nevertheless show that the policy results in a negligible (0.1%) welfare loss relative to the social optimum. This contrasts with the standard pension policy which induces the total temperature rise of 7.8°C and a social welfare loss of 5%.

Due to its environmentally superior outcome, we advocate the solution involving carbon taxes. However, the approach of conducting the climate policy through green pensions shows potential to be politically feasible. It does not significantly reduce the overall social welfare and allows the individuals to sustain relatively high consumption levels. If proven to be indeed acceptable by the public, the additional 0.3°C rise in temperature still appears to outweigh the costs of inaction.

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8 Appendix

8.1 Social planner's solution

The planner maximises

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = K_t^{\alpha} - C_t - \alpha K_t^{\alpha} - a_t \tag{15}$$

and

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t. \tag{12}$$

First-order conditions are taken with respect to the control variables C_t and a_t, and envelope theorem conditions are obtained with respect to the state variables K_t and E_t:

$$F.O.C.s$$

$$w.r.t.\ \ C_t, \qquad \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{1}{\beta C_t} \tag{38}$$

$$w.r.t.\ \ a_t, \qquad \phi\frac{\partial V_{t+1}}{\partial E_{t+1}} = \frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{39}$$

$$E.T.s$$

$$w.r.t.\ \ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \alpha\beta K_t^{\alpha-1}\left[(1-\alpha)\frac{\partial V_{t+1}}{\partial K_{t+1}} - \theta\frac{\partial V_{t+1}}{\partial E_{t+1}}\right] \tag{40}$$

$$w.r.t.\ \ E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \beta\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{41}$$

The next step involves combining the results just obtained, iterating them accordingly and eliminating marginal values. This way we reach the system of 4 difference equations in 4 unknowns which characterise the optimised model:

$$K_{t+1} = (1-\alpha)K_t^{\alpha} - C_t - a_t \tag{42}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t \tag{12}$$

$$\frac{1}{\beta C_t} = \frac{\alpha}{K_{t+1}} + \alpha K_{t+1}^{\alpha-1}\left(1 - \alpha - \frac{\theta}{\phi}\right)\frac{1}{C_{t+1}} \tag{43}$$

$$\frac{1}{\phi\beta C_t} = \frac{2\epsilon}{E_{t+1}} + \frac{1}{\phi C_{t+1}} \tag{44}$$

Rearranging the above equations yields the system given by:

$$\left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right]^{-1} = \frac{1}{\beta C_t} - \frac{2\epsilon\phi}{E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1})} \tag{45}$$

$$C_{t+1} = \left(\frac{1}{\beta C_t} - \frac{\alpha}{K_{t+1}}\right)^{-1}\left[\alpha K_{t+1}^{\alpha-1}\left(1-\alpha-\frac{\theta}{\phi}\right)\right] \tag{17}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi((1-\alpha)K_t^{\alpha} - C_t - K_{t+1}) \tag{18}$$

$$a_t = (1-\alpha)K_t^{\alpha} - C_t - K_{t+1} \tag{19}$$

By dropping the time subscripts and rearranging, we obtain the following analytic steady state solutions:

$$K^* = \left[\frac{2\alpha\beta(1-\alpha-\frac{\theta}{\phi})}{(1+\alpha\beta)}\right]^{\frac{1}{1-\alpha}} \tag{20}$$

$$a^* = \frac{\theta}{\phi}(K^*)^{\alpha} \tag{21}$$

$$C^* = (1-\alpha)(K^*)^{\alpha} - K^* - a^* \tag{22}$$

$$E^* = \left(\frac{\beta}{1-\beta}\right)2\epsilon\phi C^* \tag{23}$$

8.2 General decentralised solution

$$U_t^i = lnC_t + \beta lnC'_{t+1} + \epsilon lnE_t + \beta\epsilon lnE_{t+1} \tag{24}$$

$$C_t = T_t + (1-\tau_t^p)W_t - \tau_t^c(\theta K_t^{\alpha} - \phi a_t) - (1-\tau_t^I)I_t - (1-\tau_t^a)a_t \tag{25}$$

$$C_{t+1} = r_{t+1}I_t \tag{26}$$

$$E_{t+1} = E_t - \theta K_t^{\alpha} + \phi a_t \tag{12}$$

where transfers (taken as given) are:

$$T_t = \tau_t^p(1-\alpha)K_t^{\alpha} + \tau_t^c(\theta K_t^{\alpha} - \phi a_t) - \tau_t^I I_t - \tau_t^a a_t \tag{27}$$

To maximise (24) s.t. the constraints, we differentiate (24) with respect to I_t and a_t. This gives:

$$0 = \frac{\partial U_t}{\partial I_t} = -\frac{1-\tau_t^I}{C_t} + \frac{\beta}{I_t} \tag{46}$$

$$0 = \frac{\partial U_t}{\partial a_t} = \frac{\tau_t^c\phi - (1-\tau_t^a)}{C_t} + \frac{\epsilon\beta\phi}{E_t - \theta K_t^{\alpha} + \phi a_t} \tag{47}$$

Merging with the aggregate budget constraint and rearranging then gives:

$$a_t = \left[(1-\alpha)K_t^\alpha - \left(\frac{1-\tau_t^I+\beta}{1-\tau_t^I}\right)\frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t - \theta K_t^\alpha\right)\right]\frac{\epsilon\beta\left(1-\tau_t^I\right)}{\epsilon\beta\left(1-\tau_t^I\right)+\left(1-\tau_t^I+\beta\right)\left(1-\tau_t^a-\tau_t^c\phi\right)} \quad (48)$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^a-\tau_t^c\phi\right)\left(E_t-\theta K_t^\alpha\right) \quad (29)$$

$$I_t = \frac{\beta}{1-\tau_t^I}C_t \quad (30)$$

$$E_{t+1} = E_t - \theta K_t^\alpha + \phi a_t \quad (12)$$

Assuming $\tau_t^c > 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$, this solution can replicate the social planner's optimal plan. Therefore, we can use specific values of the steady state variables from the social planner's solution (i.e. given by (20)-(23)) and obtain consistent tax rates which ensure the above conditions are met from the following:

$$\tau^p = 1 - \beta\frac{C}{I}$$

$$\tau^c = \frac{1-\tau^p}{\phi} - \epsilon\beta\frac{C}{E}$$

8.3 Green pension solution

Individual consumption-saving decisions still follow the optimised system specified in section 4.2 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$, $\tau_t^I = 0$ & $\tau_t^p = \tau_t^a$, i.e.:

$$I_t = \beta C_t \quad (32)$$

$$C_t = \frac{1}{\epsilon\beta\phi}\left(1-\tau_t^p\right)\left(E_t - \theta K_t^\alpha + \phi a_t\right) \quad (49)$$

$$a_t = \frac{\epsilon\beta}{\epsilon\beta + \left(1+\beta\right)\left(1-\tau_t^p\right)}\left[(1-\alpha)K_t^\alpha - \left(\frac{1+\beta}{\epsilon\beta\phi}\right)\left(1-\tau_t^p\right)\left(E_t-\theta K_t^\alpha\right)\right] \quad (50)$$

When $a_t = 0$:

$$C_t = \frac{1-\alpha}{1+\beta}K_t^\alpha \quad (51)$$

$$I_t = \beta C_t \quad (52)$$

By using a series of equivalent consumption choices, the planner aims to maximise:

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^{\alpha}) + 2\epsilon lnE_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = I_t = \beta C_t \tag{32}$$

and

$$E_{t+1} = E_t + \phi\left(1 - \alpha - \frac{\theta}{\phi}\right)K_t^{\alpha} - \phi(1+\beta)C_t. \tag{33}$$

The resulting first-order and envelope theorem conditions are then given by:

$$F.O.C.$$

$$w.r.t.\ \ C_t, \qquad \frac{1}{\beta C_t} = \phi(1+\beta)\frac{\partial V_{t+1}}{\partial E_{t+1}} - \frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{53}$$

$$E.T.s$$

$$w.r.t.\ \ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \beta\phi\left(1 - \alpha - \frac{\theta}{\phi}\right)\alpha K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{54}$$

$$w.r.t.\ \ E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \beta\frac{\partial V_{t+1}}{\partial E_{t+1}} \tag{55}$$

Therefore, based on (32), (33) and (53)-(55), we can drop the time subscripts and rearrange to obtain the following steady state system:

$$K = \left[\frac{\beta(1 - \alpha - \frac{\theta}{\phi})}{1+\beta}\right]^{\frac{1}{1-\alpha}} \tag{56}$$

$$C = \frac{K}{\beta} \tag{57}$$

$$\frac{\partial V}{\partial E} = \left(\frac{\alpha}{K} + \frac{1}{\beta^2 C}\right)\frac{1}{\phi}\left[\frac{1+\beta}{\beta} - \beta\left(1 - \alpha - \frac{\theta}{\phi}\right)\alpha K^{\alpha-1}\right]^{-1} \tag{58}$$

$$E = \frac{2\epsilon}{(1-\beta)\frac{\partial V}{\partial E}} \tag{59}$$

Then, from (49) we can obtain the consistent steady state pension tax rate:

$$\tau^p = 1 - \frac{\epsilon\beta\phi C}{E - \theta K^{\alpha} + \phi a} \tag{60}$$

8.4 Standard pension solution

Individual consumption-saving decisions still follow the optimised system specified in section 4.3 by equations (28), (29), (30) and (12), although with $\tau_t^c = 0$ & $\tau_t^p = \tau_t^I = \tau_t^a$, i.e.:

$$I_t = \frac{\beta}{1-\tau_t^p} C_t \tag{61}$$

$$C_t = \frac{1}{\epsilon\beta\phi}\Big(1-\tau_t^p\Big)\Big(E_t - \theta K_t^\alpha + \phi a_t\Big) \tag{62}$$

$$a_t = \frac{\epsilon\beta}{\epsilon\beta + 1 - \tau_t^p + \beta}\left[(1-\alpha)K_t^\alpha - \left(\frac{1-\tau_t^p+\beta}{\epsilon\beta\phi}\right)\Big(E_t - \theta K_t^\alpha\Big)\right] \tag{63}$$

When $a_t = 0$:

$$C_t = \frac{1-\tau_t^p}{1-\tau_t^p+\beta}(1-\alpha)K_t^\alpha \tag{64}$$

$$I_t = \frac{\beta}{1-\tau_t^p} C_t \tag{65}$$

By using a series of equivalent consumption choices, the planner aims to maximise

$$V_t(K_t, E_t) = lnC_t + ln(\alpha K_t^\alpha) + 2\epsilon ln E_t + \beta V_{t+1}(K_{t+1}, E_{t+1}) \tag{14}$$

s.t.

$$K_{t+1} = \frac{1}{1+\epsilon}\left[\left(1-\alpha-\frac{\theta}{\phi}\right)K_t^\alpha + \frac{E_t}{\phi} - C_t\right] \tag{35}$$

and

$$E_{t+1} = \frac{1}{1+\epsilon}\left[E_t + \epsilon\phi\left(1-\alpha-\frac{\theta}{\phi}\right)K_t^\alpha - \epsilon\phi C_t\right] \tag{36}$$

The resulting first-order and envelope theorem conditions are then given by:

F.O.C.

$$w.r.t.\ \ C_t, \qquad \frac{1}{\beta C_t} = \frac{1}{1+\epsilon}\left(\frac{\partial V_{t+1}}{\partial K_{t+1}} + \epsilon\phi\frac{\partial V_{t+1}}{\partial E_{t+1}}\right) \tag{66}$$

E.T.s

$$w.r.t.\ \ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + \beta\Bigg(\frac{\partial V_{t+1}}{\partial K_{t+1}}\frac{\alpha}{1-\epsilon}\Big(1-\alpha-\frac{\theta}{\phi}\Big)K_t^{\alpha-1} + \frac{\alpha\phi}{1+\epsilon}\Big(\epsilon(1-\alpha)-\frac{\epsilon\theta}{\phi}\Big)K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial E_{t+1}}\Bigg) \tag{67}$$

$$w.r.t.\ \ E_t, \qquad \frac{\partial V_t}{\partial E_t} = \frac{2\epsilon}{E_t} + \frac{\beta}{1+\epsilon}\Bigg(\frac{1}{\phi}\frac{\partial V_{t+1}}{\partial K_{t+1}} + \epsilon\frac{\partial V_{t+1}}{\partial E_{t+1}}\Bigg) \tag{68}$$

Then, we consider the system of (35), (36) and (66)-(68) which we solve numerically. The optimal steady state pension tax rate can then be inferred from:

$$\tau^p = 1 - \beta(1+\epsilon)\left(\frac{C}{1-\alpha-\frac{\theta}{\phi}}\right)K^\alpha + \frac{E}{\phi - C} \tag{69}$$

8.5 Initial level of capital

$$V_t(K_t) = lnC_t + ln(\alpha K_t^\alpha) + \beta V_{t+1}(K_{t+1}) \tag{70}$$

s.t.

$$K_{t+1} = K_t^\alpha - C_t - \alpha K_t^\alpha \tag{71}$$

We differentiate to obtain the following first-order and envelope theorem conditions:

F.O.C.

$$w.r.t.\ \ C_t, \qquad \frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{1}{\beta C_t} \tag{72}$$

E.T.

$$w.r.t.\ \ K_t, \qquad \frac{\partial V_t}{\partial K_t} = \frac{\alpha}{K_t} + (1-\alpha)\alpha\beta K_t^{\alpha-1}\frac{\partial V_{t+1}}{\partial K_{t+1}} \tag{73}$$

what eventually yields the following steady state:

$$K^* = \left(\frac{2\alpha\beta(1-\alpha)}{1+\alpha\beta}\right)^{\frac{1}{1-\alpha}} \tag{74}$$

$$C^* = \Big((1-\alpha)(K^*)^{\alpha-1} - 1\Big)K^* \tag{75}$$